6.4 exponential growth, decay

1.1 1.2 1.3 RAD AUTO REAL

Suppose you deposit $800 in an account that pays 6.3% annual interest. How much will you have 8 years later if the interest is compounded:

a) annually, b) quarterly, c) monthly
d) daily, e) continuously

1304.24 ($k=1$), 1319.07 ($k=4$), 1322.95 ($k=12$)

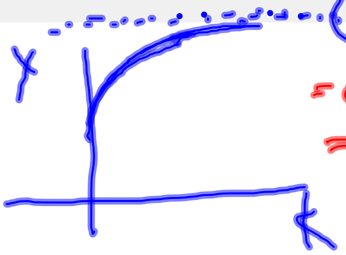
$k=365$: 1324.21

$k=??\ \infty$: 1324.26

$y=y_0e^{rt}$

$=e^{.063\cdot 8}$

$=1324.26$



$$y = FV = \overset{y_0}{PV}\left(1+\frac{r}{k}\right)^{k\cdot t}$$

FV = future value

PV = present value

t = time (years)

r = interest rate (decimal)

k = # times compounded per year

$y = y_0 e^{rt}$

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differential equation for exponential growth and decay

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{dt} = -ky$$

y = amount

k = growth constant or decay constant

$$\boxed{\text{half life} = \frac{\ln 2}{k}}$$

$\frac{dy}{dt}$ = how fast the amount increases, decreases

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Solve $\frac{dy}{dt} = ky$

solution

seperate variables

$$\int \frac{dy}{y} = \int k \, dt$$

$$\ln|y| + c_1 = kt + c_2$$

$$e^{\ln|y|} = e^{(kt + c)}$$

$$|y| = e^{kt+c}$$

$$y = e^{kt} \cdot e^{c}$$

$$y = c_4 e^{kt}$$

use initial condition

$t=0 \quad y = y_0$

$$c_4 = y_0$$

$$\boxed{y = y_0 e^{kt}}$$

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How long will \$7000 take to double at 5%?

How long will \$17,000 take to double at 5%?

(compounded continuously)

$$\frac{14{,}000}{7000} = \frac{7000}{7000} e^{.05t}$$

$$2 = e^{.05t}$$

$$\ln(2) = .05t \ln(e)$$

$$\ln(e) = 1$$

$$\frac{\ln(2)}{.05} = \frac{.05t}{.05}$$

$$\frac{\ln(2)}{.05} = t$$

$$t = 13.8629$$

$$\boxed{\text{doubling time} = \frac{\ln 2}{k}}$$

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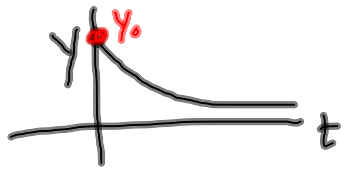
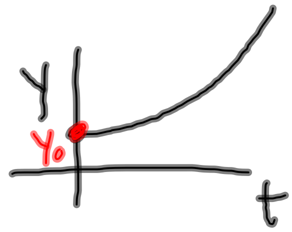
decay $y = y_0 e^{-kt}$



growth $y = y_0 e^{kt}$



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A bacteria colony begins with 700 bacteria. It doubles every 4.5 hours. How many bacteria will there be in 75 hrs?

7.2×10^7

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